

Eden Primary Calculation Policy - January 2014

Introduction

This policy outlines the progression we will teach at Eden primary through the written strategies for addition, subtraction, multiplication and division. Children work through the progression so that they can understand, use, apply and explain formal methods of calculation by the end of Year 6, although it is expected that the children will be able to explain and apply the current mental and written methods they are using.

We would expect the majority of each class to be working at age-appropriate levels as set out in the National Curriculum 2014. However, children should not be made to go onto the next stage if they are not ready or confident and teachers must present strategies and equipment appropriate to the children's level of understanding. Where children master concepts quickly, they should be given investigations and complex problems to solve.

This policy also includes examples and diagrams, showing how we teach calculations as consistency in layout and presentation is important to support learning and progression. The policy also includes the equipment and resources that will be used to support children's understanding of each strategy.

The Importance of Mental Maths

Whilst this policy focuses on written calculations in maths, we recognise the importance of mental strategies and known facts that form the basis of all calculations.

- Children should be encouraged to consider if a mental calculation would be appropriate before using written methods.
- They should be encouraged to approximate before calculating and check whether their answer is reasonable.
- It is vital that children understand the = sign means *is equivalent to*, not *makes*, and that children see calculations where the equals sign is in different positions, e.g. $3 + 2 = 5$ and $5 = 3 + 2$.

The following checklist outlines the key skills and number facts that children are expected to develop throughout the school. See *Teaching mental calculation strategies to level 5 – QCA (2004)* for further details

To add and subtract successfully, children should be able to:

- Recall all addition pairs to 9+9 and number bonds to 10
- Count on or back in repeated steps of 1, 10, 100, 1000
- Recognise addition and subtraction as inverse operations and use this to solve calculations (e.g. $15 + 7 = 22$, $22 - 15 = 7$)

- Add mentally a series of one digit numbers (e.g. $5+8+4$)
- Add and subtract multiples of 10 or 100 using the related addition fact and their knowledge of place value (e.g. $600+700$, $160 - 70$)
- Add or Subtract the nearest multiple of 10, 100 and 1000 and adjust (e.g. $24 - 19 = 24 - 20 + 1 = 5$, $142+49 = 142+50-1$)
- Partition 2 and 3 digit numbers into multiples of 100, 10 and 1 in different ways (e.g. $74 = 70 + 4$ or $60+14$)
- Use estimation by rounding to check answers are reasonable.

To multiply and divide successfully, children should be able to

- Recall multiplication facts to $12 \times 12 = 144$ and division facts to $144 \div 12 = 12$ (Y2: 2x, 5x, 10x, Y3: 3x, 4x, 8, Y4: up to 12×12)
- Apply the knowledge of doubles and halves to known facts. e.g. 8×4 is double 4×4
- Use factors to support multiplication (e.g. $8 \times 12 = 8 \times 4 \times 3$)
- Know the outcome of multiplying by 0 and by 1 and of dividing by 1.
- Understand the effect of multiplying and dividing whole numbers by 10, 100 and later 1000
- Derive other results from multiplication and division facts and multiplication by 10, 100 or 1000 (e.g. e.g. If I know $3 \times 7 = 21$, what else do I know? $30 \times 7 = 210$, $300 \times 7 = 2100$, $3000 \times 7 = 21000$, $0.3 \times 7 = 2.1$ etc.)
- Use closely related facts already known(e.g. $13 \times 11 = (13 \times 10) + (13 \times 1) = 130 + 13 = 143$

MANY MENTAL CALCULATION STRATEGIES WILL CONTINUE TO BE USED. THEY ARE NOT REPLACED BY WRITTEN METHODS.

Chapter 1: PROGRESSION THROUGH COUNTING

The importance of counting cannot be over stressed. Children need to be confident in the following areas. Teachers need to focus on this in the early years but need to revisit counting with older children to ensure the skills are secure.

- a) One to one correspondence Children synchronise their counting and pointing, keeping track of their counting as they go, assigning one number name to one object and only counting each object once. Counting static pictures is harder and children need to develop a system to know which they have counted as they go along
- b) Stable order of counting Children need to know that the list of words used is in a repeatable order. This stable list is at least as long as the number of items to be counted. The lists used will change as the children grow older e.g. to 100, in 3s, involving negative numbers, decimals etc
- c) Cardinal aspect of number The last number counted is the number of objects in the set.
- d) Abstract principle of number Children count things which cannot be moved or touched such as sounds, imaginary objects or even words.

Greater than / Less than/Equivalent to

Children use direct comparison



Encourage children to discuss how to place quantities in order. How do they know they are in order. Is there another order they could be put in?

Encourage children to develop their own ways of recording as well as introducing formal symbols.

Use weighted manipulatives such as Cuisenaire rods, Numicon and Diennes blocks with balance scales to compare

Chapter 2: PROGRESSION THROUGH CALCULATIONS FOR ADDITION

Children should understand that addition is commutative and therefore calculations can be rearranged, e.g. $4 + 13 = 17$ is the same as $13 + 4 = 17$.

YR

1) Counting two sets (aggregation)

Children need to explore addition as putting together – two or more amounts or numbers are put together to make a total.

They should explore this with a wide range of objects including objects of mixed sets (*e.g. cows, sheep - how many animals?*), counters, multilink etc

They need to be taught to count one set, then the other, put them together and then count again starting at 1.

Use the bead string – count out the two sets and then draw them together and count the total.

2) Counting two sets (augmentation or counting on)

Children need lots of play experiences to prepare them for this stage – count one set, then hide it (*teddy has eaten 3 biscuits, there are 4 on the plate, how many were there to start with?*)

Count one set, then count on from the total of the first; 7 ... 8...9...10...11...12

Using objects, multilink, bead strings

Numicon Model

Place two shapes together and match with the corresponding shape. (*Children need lots of practice so that they recognise all the Numicon shapes automatically*)



Recording

Children record their answers with pictures or their own representations.

They use number and symbol cards to make number sentences

Children **who are ready** may record this as:

$$6 = 2 + 4 \quad 6 = 3 + 3 \quad 6 = 4 + 2 \quad 6 = 0 + 6 \quad 6 = 1 + 5 \quad 6 = 5 + 1$$

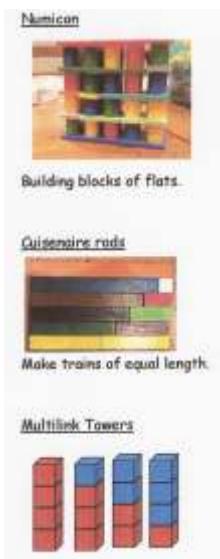
Key Questions to assess understanding

How does the child approach this? Do they understand the concept of total, altogether? Do they touch or move the objects as they count? Are the counting skills secure? Can they describe what they are doing and why? Can they articulate what the final number means? Do they start from the first set total and count on or do they count up to the start number from 1? What mathematical word do we use when put these sets together? (Add) Can they work out the answer when you replace one of the numbers with a numeral?

Y1

- a) Children will initially use practical equipment to combine groups of objects to find the total as in Reception.

b) Partitioning to 20



It is vital that the children are confident in partitioning numbers in a variety of ways - ensuring that the children can think flexibly about numbers. It can be explored through a wide variety of manipulatives.

All the manipulatives can represent a range of numbers and decimals as the children's understanding of the number system develops. e.g. all complements to 1, 10, 100 etc.; partitioning numbers in all possibilities 9, 0 + 9, 1 + 8, 2 + 7, 3 + 6, 4 + 5; an infinite number of decimal combinations.

Always encourage 'All Possibilities' (systematic working and how do you know you have them all?)

c) Bridging through 10

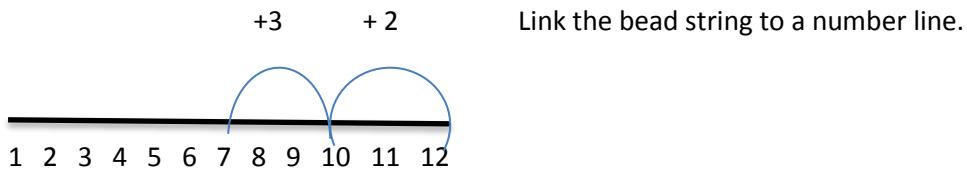
It is crucial that the children start to think flexibly about the numbers they are using. Not just counting but calculating. You need to bridge the children between the manipulative bead bar and an image of a bead bar.

i) Bead String Model

$$7 + 5$$



How many more to the next multiple of 10? 3 If we use 3 of the 5 to get to 10, how many more do we need to add on?

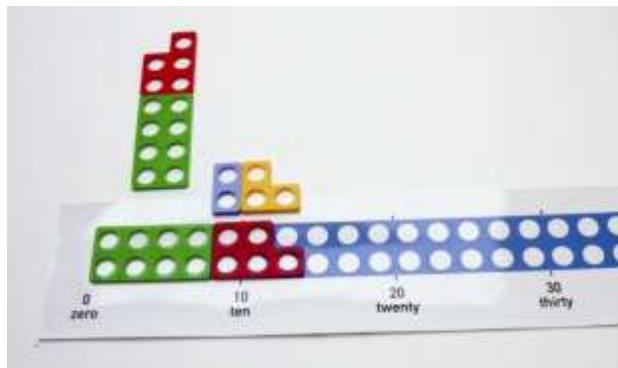


Ensure that you are prompting the children to visualise the bead string in their head as well

– this is essential to ensure that the children are thinking carefully about the numbers that they are working with and considering different strategies from an early age, they will then have an idea of the answer prior to completing the calculation.

ii) Numicon Model

Use Numicon to provide the children with an alternative image for bridging through 10.



$8+5=\dots$ what do we need to add to 8 to make 10 (2) How many does that leave us... $3\dots 10+3=13$

Using a number track. Place on the first value in Numicon shapes, can they see the shape that would complete to the next multiple of 10. What Numicon piece do they need to make the second Numicon shape. Place this on the number line to calculate the total.

Key Questions

Do the children know their number bonds to 10? Can they read and write numbers to 20? Do they have a secure understanding of the number system to 20? Can they use visualisation to solve problems without concrete apparatus? Can they extend their knowledge of numbers to 10 to solve problems with nos to 20?

Recording

Use of number line

Number sentences e.g. $6 = 2 + 4$ $3+3=6$ $7+5=12$

Y2

- a) Children will continue to use the bead string/number line to bridge through 10 to add TO + O, TO +T, O+O+O

b) Children will transition to Dienes to support their calculations.

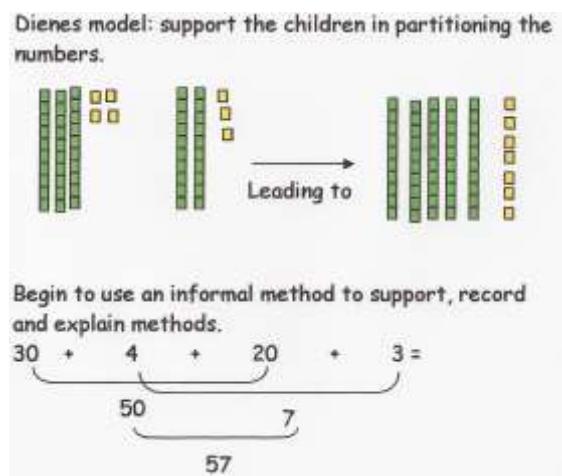
Ideas for familiarising the children with Dienes

- Link it with the number line image introduced above.
- Use on number tracks, placing a single unit on each space
- Use it in place value charts
- Allow the children to explore the relationship between the units and tens etc. Can they discover the pattern?
- Let them explore the equivalence (balance scales and direct comparison)

c) Partitioning to 100 (using the same partitioning models as in year 1)

d) TO + TO

(I) **Aggregation:** Children combine the two sets, ones with ones and tens with tens, starting with ones. E.g $34+23=$



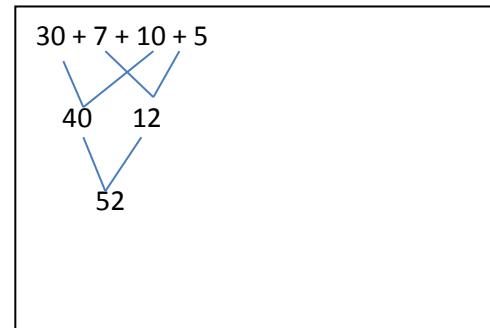
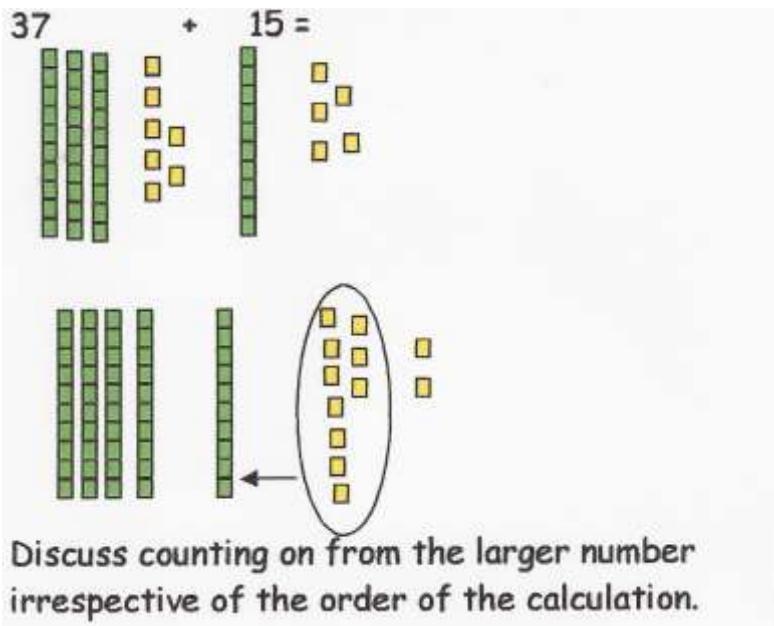
When confident:

- i) **Augmentation:** Encourage children to begin to count out from the first set of units and tens, avoid starting at 1 i.e. $34+23=34...44...54...55...56...57$. Discuss counting on from the larger number irrespective of the order of the calculation.

E) Beginning Exchange.

Once secure in partitioning for addition, children to begin to explore exchanging. When the units total more than 10, children should be encouraged to exchange 10 ones for 1 ten. This is the start of children understanding 'carrying' in vertical addition.

Introduce the term 'exchange'. Allow lots of time exchanging ones (units) for single 10s block (ensure the children understand that they are equivalent—but not the same).



F) Compensation (Adding 9 or 11)

Give children lots of opportunities to use bead string to explore adding 9 or 11 to lots of different start numbers. Ask questions such as how can we make it more efficient? How can adding 10 help?
Move on to annotating on the image of a bead bar and then the number line.

Key Questions

Do children make the link between Dienes, partitioning, recording and the expanded vertical method?

Recording

Initially they will record the calculations using their own drawings of the Base 10 equipment (as lines for the 10 rods and dots for the ones)

Move on to informal recording

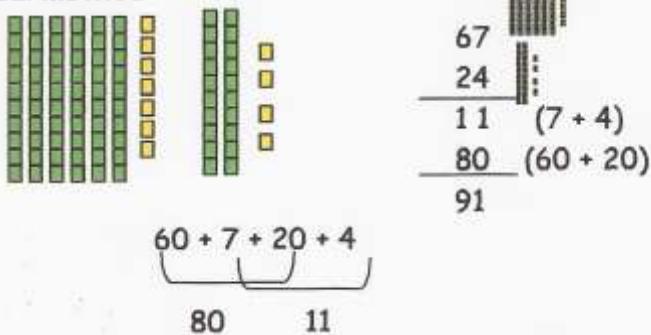
Children **who are ready** may begin to record using formal written method of column addition

Y3

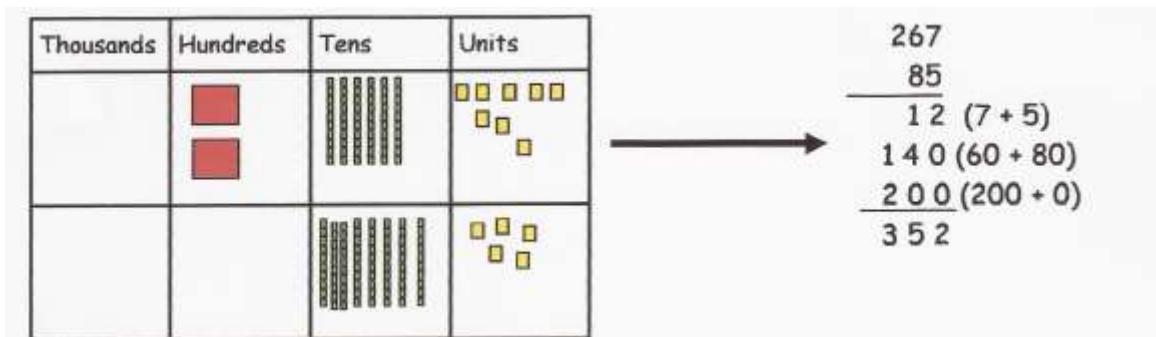
- Children will begin to use informal pencil and paper methods (jottings) to support, record and explain partial mental methods building on existing mental strategies.
- Children continue to develop their knowledge of number bonds, visualisation of key manipulatives in order to be able to add mentally HTO + O, HTO + T, HTO + H
- Children will build on their knowledge of using Base 10 equipment from Y2 and continue to use this to support with the transition into a **formal vertical method**. They will calculate with numbers up to 3 digits.

Children should add the **least significant digits** first as preparation for the compact method.

Ensure that the children make the link between dienes, partitioning recording and the expanded vertical method

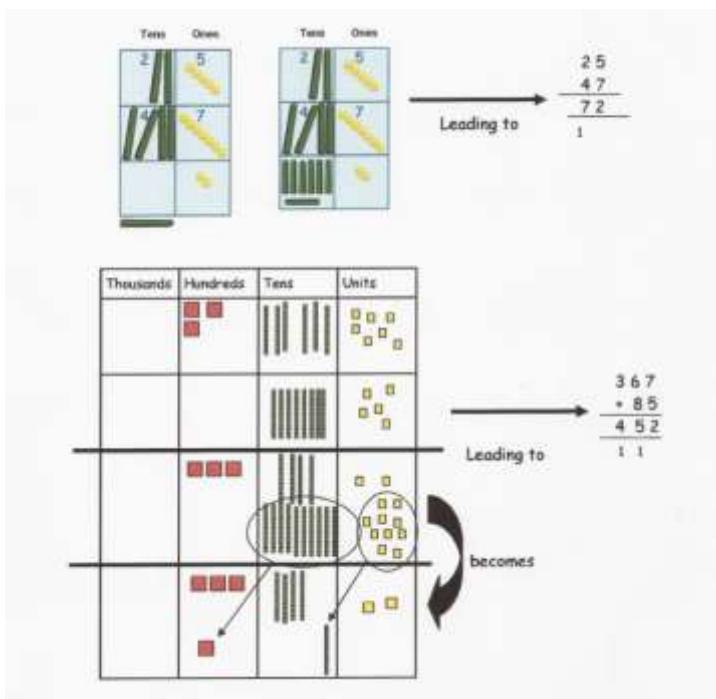


NB Numbers in brackets are for exemplification by teachers – children are not expected to write these in their answers.



Y4

- a) **Compact written method** - Children will continue to refine the model, for numbers up to four digits, compacting the recording by carrying below the line.



Place value counters and money can be used instead of Dienes as children's conceptual understanding develops

Using similar methods, children will continue to develop and refine the compact method. They will

- Add several numbers with different numbers of digits;
- Begin to add two or more three-digit sums of money, with or without adjustment from the pence to the pounds;
- Know that the decimal points should line up under each other, particularly when adding or subtracting mixed amounts, e.g. £3.59 + 78p.

Y5

- a) Children should select the appropriate method (mental or written) to solve addition problems using increasingly large numbers
- b) Children should extend the carrying method to numbers with at least four digits.

The image shows three examples of vertical addition with carrying:

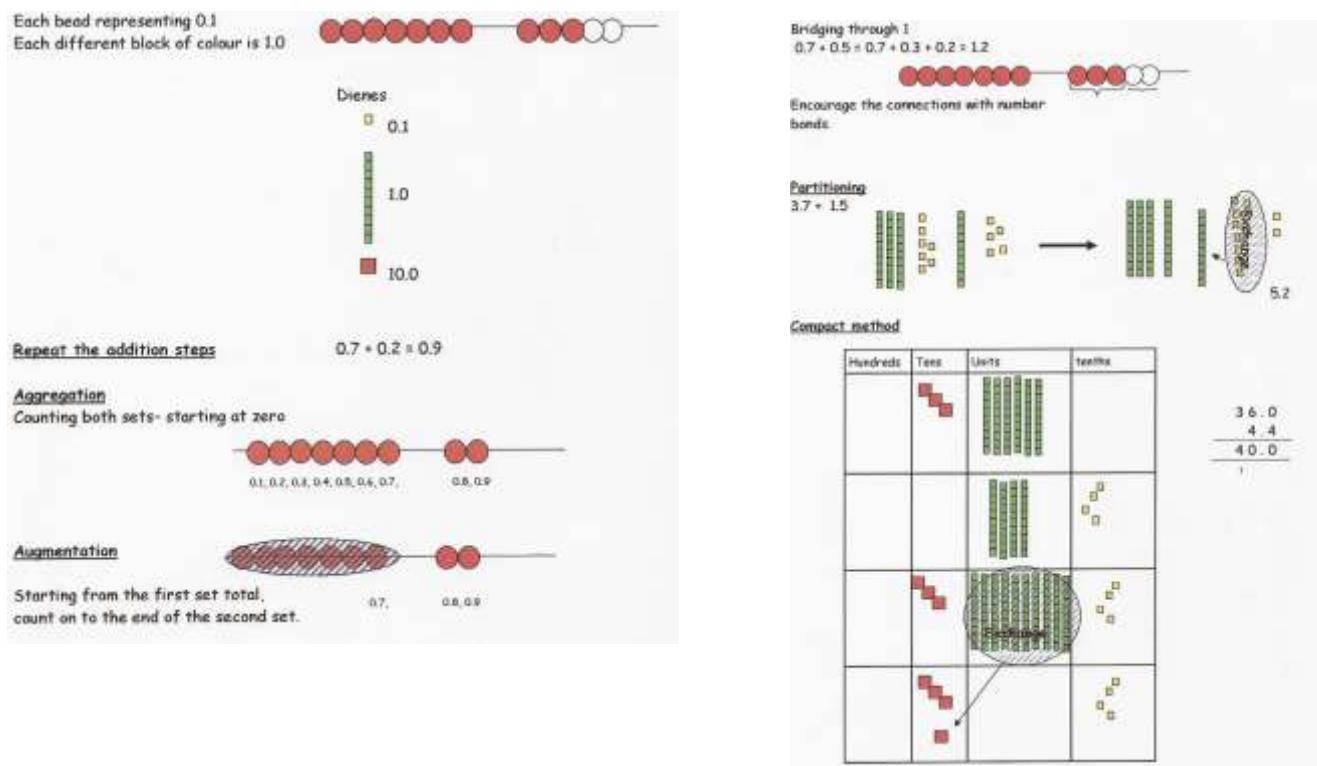
- Example 1:
$$\begin{array}{r} 587 \\ + 475 \\ \hline 1062 \end{array}$$
- Example 2:
$$\begin{array}{r} 3587 \\ + 675 \\ \hline 4262 \end{array}$$
- Example 3:
$$\begin{array}{r} 3121 \\ + 37 \\ \hline 148 \\ \hline 3306 \end{array}$$

Using similar methods, children will:

- add several numbers with different numbers of digits;
- begin to add two or more decimal fractions with up to three digits and the same number of decimal places;
- know that decimal points should line up under each other, particularly when adding or subtracting mixed amounts, e.g. 3.2 m + 280 cm.

C) Addition of Decimals

Return to the models the pupils are familiar with. Ensure they are confident with counting in decimals, both forwards and backwards.



Y6

Children should extend the carrying method to number with any number of digits.

$$\begin{array}{r}
 65.84 \\
 + 58.48 \\
 \hline
 124.32
 \end{array}
 \quad
 \begin{array}{r}
 42 \\
 6432 \\
 786 \\
 \hline
 11944
 \end{array}$$

Using similar methods, children will

- add several numbers with different numbers of digits;
- begin to add two or more decimal fractions with up to four digits and either one or two decimal places;
- know that decimal points should line up under each other, particularly when adding or subtracting mixed amounts, e.g. $401.2 + 26.85 + 0.71$.

By the end of year 6, children will have a range of calculation methods, mental and written. Selection will depend upon the numbers involved.

Chapter 3: PROGRESSION THROUGH CALCULATIONS FOR SUBTRACTION

It is essential that children are provided with different models for subtraction right from the start as opposed to only being introduced to the ‘take away’ method.

Children need lots of experience counting both forward and backwards, starting from different starting points.

Make sure children realise that subtraction is not commutative (cannot be done in any order) 7-5 is not the same as 5-7.

YR

1) Removing Items from a set (reduction or taking away)

Children need to explore subtraction as taking away some items from a set to leave a total. They should explore this with a wide range of objects including objects of mixed sets, counters, multilink etc. (e.g. Jane had 5 sweets and ate 2 of them. How many were left?)

They need to be taught to count the total, remove the amount and then count how many remain.

Use the bead string – count out the total, draw out the amount and then count the remainder.

This model moves easily onto counting back on a number line.

2) Comparing two sets (comparison or difference)

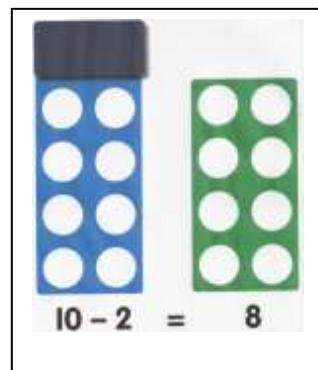
Compare two sets. How many more are there in the larger one. (Useful when the two numbers are close together) (e.g. There were 4 boys and 2 girls. How many more boys than girls were there?)

Using objects, multilink, bead strings

Numicon Model

Start with your original shape, take away the corresponding shaped Numicon cover. What number remains?

Alternatively, compare two Numicon shapes in the same way. What is the difference between the two numbers



Recording

Children record their answers with pictures or their own representations.



They use number and symbol cards to make number sentences

Children **who are ready** may record this as:

$$6 - 2 = 4 \quad 3 = 6 - 3$$

Key Questions to assess understanding

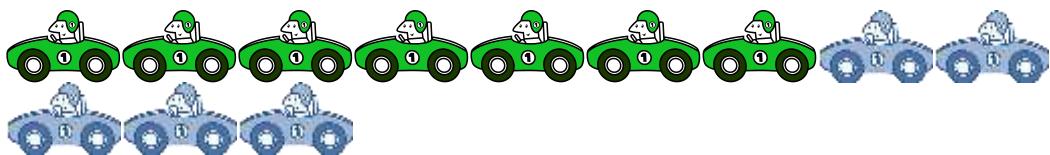
How does the child approach this? Do they understand the concept of subtraction as taking away and difference? Do they touch or move the objects as they count? Are the counting skills secure? Can they describe what they are doing and why? Can they articulate what the final number means? Can they work out the answer when you replace one of the numbers with a numeral?

Y1

- a) Children will initially use practical equipment to subtract groups of objects to find the total as in Reception.

b) Partitioning

Children relate their growing understanding of partitioning (see Addition Chapter) to introduce a third model for subtraction.



12 is made up of 7 and 5

Therefore $5+7=12$, $7+5=12$, $12-5=7$,

e.g. In a box there are 12 cars, 7 are Tom's and the rest are Noah's. How many does Noah have?

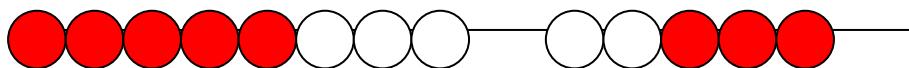
c) Bridging through 10

It is crucial that the children start to think flexibility about the numbers they are using. Not just counting but calculating. You need to bridge the children between the manipulative bead bar and an image of a bead bar, just as for addition.

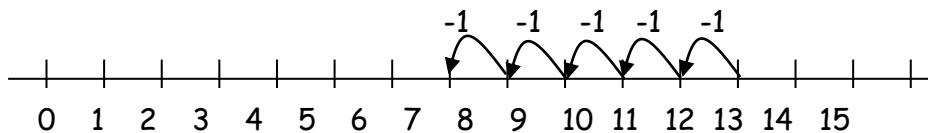
i) Bead String Model

Bead strings or bead bars can be used to illustrate subtraction including bridging through ten by counting back 3 then counting back 2.

$$13 - 5 = 8$$



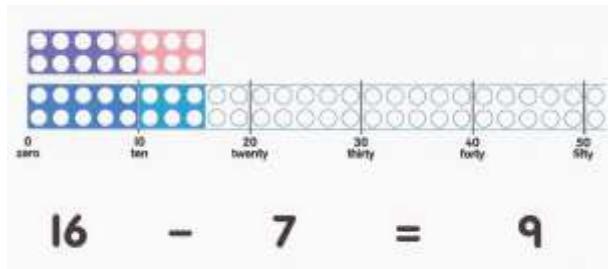
Then link the beads with a number line.



Ensure that you are prompting the children to visualise the bead string in their head as well – this is essential to ensure that the children are thinking carefully about the numbers that they are working with and considering different strategies from an early age, they will then have an idea of the answer prior to completing the calculation.

ii) Numicon Model

Use Numicon to provide the children with an alternative image for bridging through 10.



16-7.... How many do we need to take to get us to 10? (6).. How many more do we need to take away? (1). So how many do we have left? 9

Using a number track. Place on the first value in Numicon shapes, can they see the shape that would complete to the previous multiple of 10. What Numicon piece do they need to make the second Numicon shape. Place this on the number line to calculate the total.

Key Questions

Do the children know their number bonds to 10? Can they read and write numbers to 20? Do they have a secure understanding of the number system to 20? Can they use visualisation to solve problems without concrete apparatus? Can they extend their knowledge of addition to solve problems related to subtraction?

Recording

Use of number line Number sentences e.g. 13-8=5 12=14-2

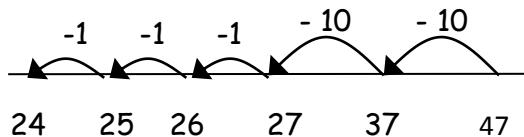
Y2

- A) Children will continue to use the bead string/number line to bridge through 10 to subtract TO - O, TO -T, TO-TO (where appropriate)

i) **Counting back**

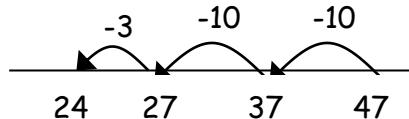
- ✓ First counting back in tens and ones.

$$47 - 23 = 24$$



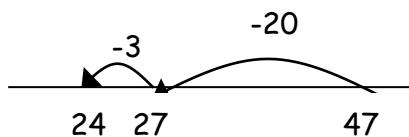
- ✓ Then helping children to become more efficient by subtracting the units in one jump (by using the known fact $7 - 3 = 4$).

$$47 - 23 = 24$$



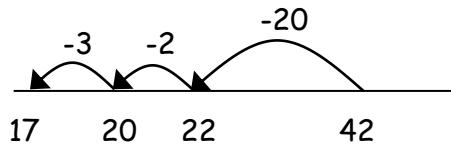
- ✓ Subtracting the tens in one jump and the units in one jump.

$$47 - 23 = 24$$



- ✓ Bridging through ten can help children become more efficient.

$$42 - 25 = 17$$



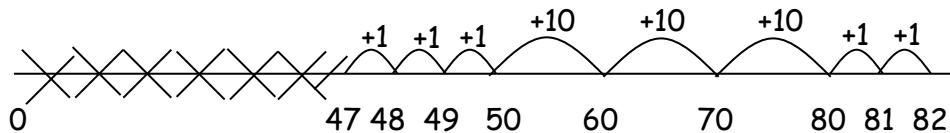
ii) Counting on

If the numbers involved in the calculation are close together or near to multiples of 10, 100 etc, it can be more efficient to count on.

Count up from 47 to 82 in jumps of 10 and jumps of 1.

The number line should still show 0 so children can cross out the section from 0 to the smallest number. They then associate this method with 'taking away'.

$$82 - 47$$



Help children to become more efficient with counting on by:

- ✓ Subtracting the units in one jump;
- ✓ Subtracting the tens in one jump and the units in one jump;
- ✓ Bridging through ten.

B) Partitioning to 100 (using the same partitioning models as in year 1)

C) TO – TO (children should be taught with quantities that do NOT require exchange)

Children use PV cards to support partitioning and Dienes to create the first number. They take away the second and count the remainder. REINFORCE STARTING FROM THE LEAST SIGNIFICANT DIGIT TO PREVENT ERRORS WHEN DECOMPOSITION IS INTRODUCED.

When solving the calculation $89 - 57$, children should know that 57 does NOT EXIST AS AN AMOUNT it is what you are subtracting from the other number. Therefore, when using base 10 materials, children would need to count out only the 89.

$$\begin{array}{r}
 89 \\
 - 57 \\
 \hline
 \end{array}
 \qquad
 \begin{array}{r}
 80 + 9 \\
 50 + 7 \\
 \hline
 30 + 2 = 32
 \end{array}$$

D) Compensation (Subtracting 9 or 11)

Give children lots of opportunities to use bead string to explore subtracting 9 or 11 to lots of different start numbers. Ask questions such as how can we make it more efficient? How can subtracting 10 help? Move on to annotating on the image of a bead bar and then the numberline.

$$30 - 11 \quad 30 - 10 - 1$$

$$20 - 9 \quad 20 - 10 + 1$$

Key Questions

Do children make the link between Dienes, partitioning, recording and the expanded vertical method?

Recording

Initially they will record the calculations using their own drawings of the Base 10 equipment (as lines for the 10 rods and dots for the ones)

Move on to informal recording

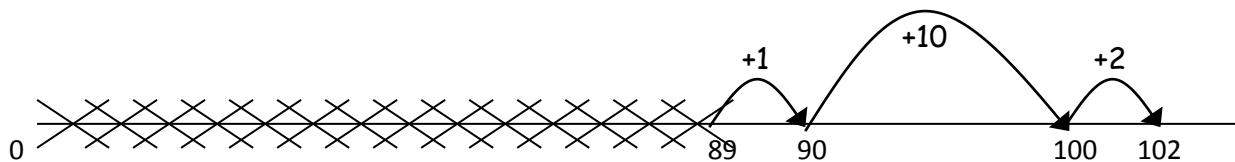
Children **who are ready** may begin to record using formal written method of column subtraction.

Y3

- Children will begin to use informal pencil and paper methods (jottings) to support, record and explain partial mental methods building on existing mental strategies.
- Children continue to develop their knowledge of number bonds, visualisation of key manipulatives in order to be able to add mentally HTO - O, HTO - T, HTO - H
- Children will build on their knowledge of using Base 10 equipment from Y2 and continue to use this to support with the transition into a formal vertical method. They will calculate with numbers up to 3 digits.

e.g. Where the numbers are involved in the calculation are close together or near to multiples of 10, 100 etc. counting on using a number line should be used.

$$102 - 89 = 13$$



d) Partitioning and decomposition

Once secure in partitioning for subtraction, children to begin to explore exchanging. When the number of units to be taken away is larger than the original units total then exchange must occur. Children should be encouraged to exchange 10 ones for 1 ten. This is the start of children understanding 'exchanging' or 'decomposition' in vertical subtraction.

Introduce the term 'exchange'. Allow lots of time exchanging ones (units) for single 10s block (ensure the children understand that they are equivalent—but not the same).

From this the children will begin to exchange.

$$\begin{array}{r} 71 = \text{Step} & 70 + 1 \\ - 46 & - 40 + 6 \\ \hline \end{array}$$

Step 2 $60 + 11$
 $- 40 + 6$
 $20 + 5 = 25$

The calculation should be read
as e.g. take 6 from 1.

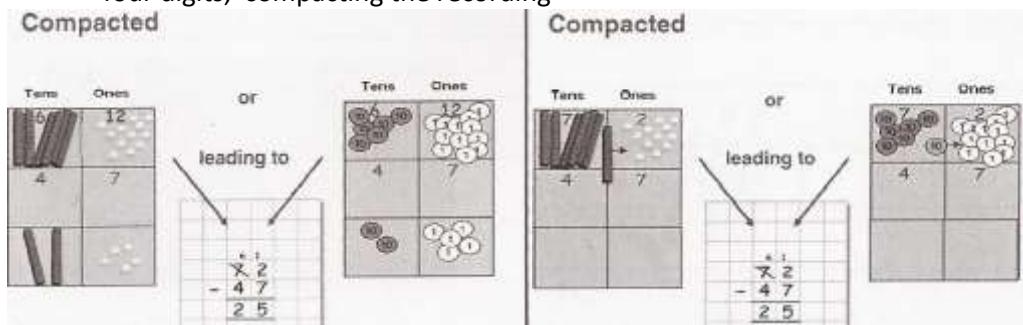
This would be recorded by the children as

$$\begin{array}{r}
 60 \\
 \cancel{70} + 1 \\
 - 40 + 6 \\
 \hline
 20 + 5 = 25
 \end{array}$$

Children should know that units line up under units, tens under tens, and so on.

Y4

- a) Children should select the appropriate method (mental or written) to solve addition problems using increasingly large numbers
- b) Children should extend formal written methods to numbers with up to 4 digits.
- c) **Compact written method** - Children will continue to refine the model, for numbers up to four digits, compacting the recording



Children should:

- ✓ be able to subtract numbers with different numbers of digits;
- ✓ using this method, children should also begin to find the difference between two three-digit sums of money, with or without 'adjustment' from the pence to the pounds;
- ✓ know that decimal points should line up under each other.

For example:

$$\begin{array}{rcl}
 \text{£8.95} & = & 8 + 0.9 + 0.05 & \text{leading to} \\
 - \underline{\text{£4.38}} & & - 4 + 0.3 + 0.08 & \\
 \\
 & = & 8 + 0.8 + 0.15 & (\text{adjust from T to U}) & 8.95 \\
 & & - 4 + 0.3 + 0.08 & & - \underline{4.38} \\
 & & 4 + 0.5 + 0.07 & & \\
 \\
 & & = \text{£4.57} & &
 \end{array}$$

Alternatively, children can set the amounts to whole numbers, i.e. $895 - 438$ and convert to pounds after the calculation.

NB If the children have reached the concise stage they will then continue this method. They will not go back to using the expanded methods.

Y5

- Children should select the appropriate method (mental or written) to solve addition problems using increasingly large numbers
- Children should extend the carrying method to numbers with at least four digits.
- Subtraction of Decimals** - Return to the models the pupils are familiar with. Ensure they are confident with counting in decimals, both forwards and backwards. (see Addition)

A handwritten subtraction problem:
$$\begin{array}{r} 6784 \\ - 286 \\ \hline 468 \end{array}$$

The calculation shows regrouping from the tens column to the ones column. The 8 in 6784 is crossed out and a 1 is written above it, indicating a borrow of 10. The 4 in 6784 is then decreased by 8, resulting in a 6. The 2 in 286 is then decreased by 6, resulting in a 6. The final result is 468.

Children should:

- ✓ be able to subtract numbers with different numbers of digits;
- ✓ begin to find the difference between two decimal fractions with up to three digits and the same number of decimal places;
- ✓ know that decimal points should line up under each other.

Y6

Decomposition

A handwritten subtraction problem using decomposition:
$$\begin{array}{r} 54.67 \\ - 26.84 \\ \hline 37.83 \end{array}$$

The calculation shows regrouping from the tenths column to the hundredths column. The 6 in 54.67 is crossed out and a 1 is written above it, indicating a borrow of 10. The 4 in 54.67 is then decreased by 6, resulting in a 3. The 6 in 54.67 is then decreased by 1, resulting in a 3. The 2 in 26.84 is then decreased by 3, resulting in a 3. The final result is 37.83.

Children should:

- ✓ be able to subtract numbers with different numbers of digits;
- ✓ be able to subtract two or more decimal fractions with up to three digits and either one or two decimal places;
- ✓ know that decimal points should line up under each other.

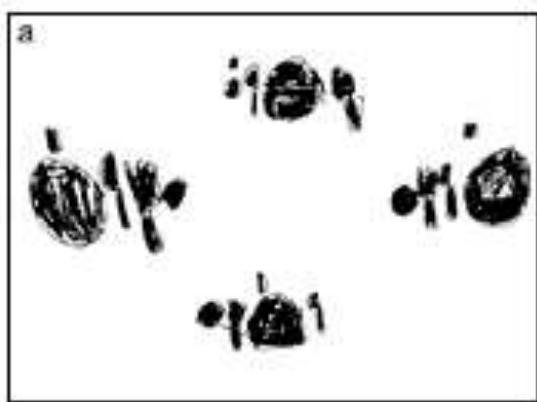
By the end of year 6, children will have a range of calculation methods, mental and written.

Selection will depend upon the numbers involved.

Chapter 4: PROGRESSION THROUGH CALCULATIONS FOR MULTIPLICATION

YR and Y1

Children will have real practical experiences of handling equal groups of objects and will count in 2s, 5s and 10s. They will work on practical problem solving activities involving equal sets or groups.



Y2

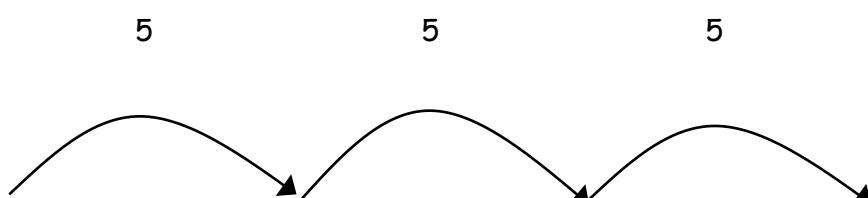
Children will develop their understanding of multiplication and use jottings to support calculation. They will record them formally using the notation $4 \times 3 = 12$

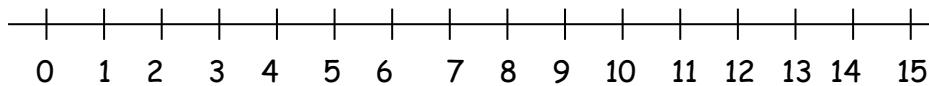
✓ **Repeated addition**

3 times 5 is $5 + 5 + 5 = 15$ or 3 lots of 5 or 5×3

Repeated addition can be shown easily on a number line:

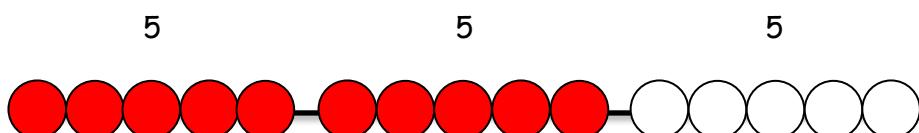
$$5 \times 3 = 5 + 5 + 5$$





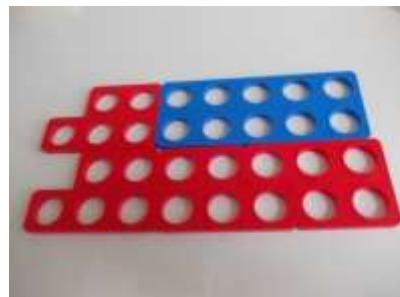
and on a bead bar:

$$5 \times 3 = 5 + 5 + 5$$



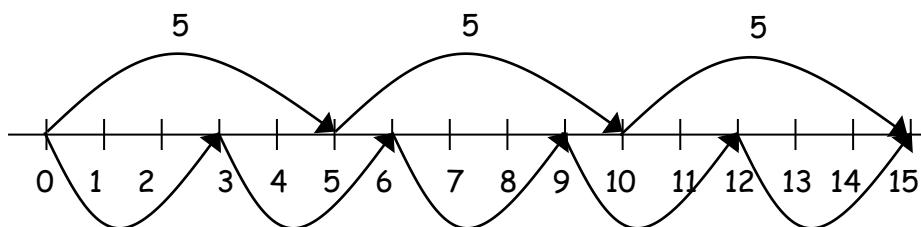
Cuisenaire Rods

Children will also learn to partition totals into equal trains using Cuisenaire rods or Numicon



✓ Commutativity

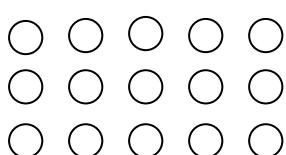
Children should know that 3×5 has the same answer as 5×3 . This can also be shown on the number line.



3 3 3

✓ Arrays

Children should be able to model a multiplication calculation using an array. This knowledge will support with the development of the grid method. It also supports finding the factors of a number.



$$5 \times 3 = 15$$

$$3 \times 5 = 15$$

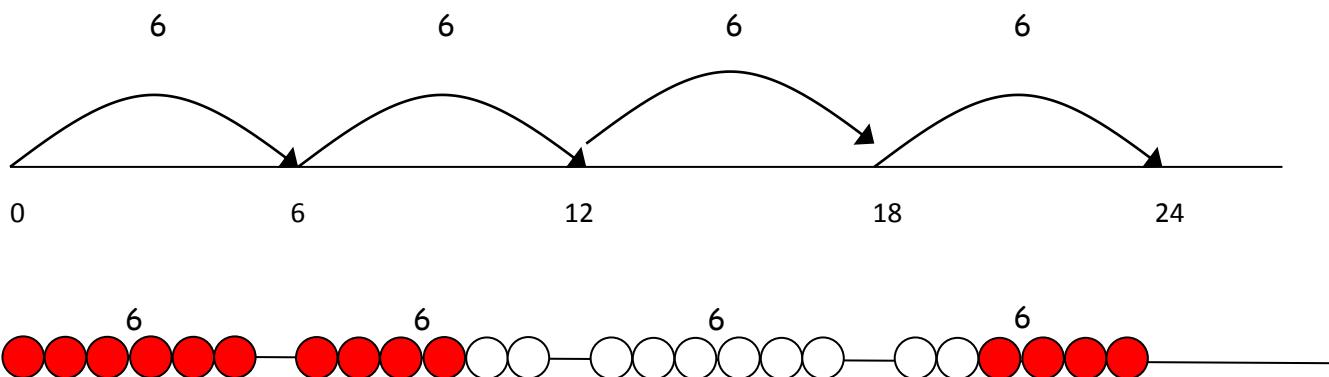
Y3

Children will continue to use:

✓ **Repeated addition**

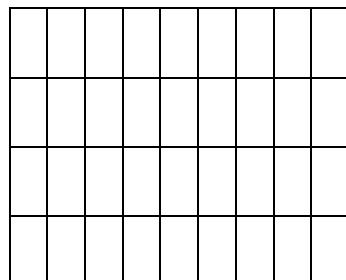
4 times 6 is $6 + 6 + 6 + 6 = 24$ or 4 lots of 6 or 6×4

Children should use number lines or bead bars to support their understanding.



✓ **Arrays**

Children should be able to model a multiplication calculation using an array. This knowledge will support with the development of the grid method.



$$9 \times 4 = 36$$

✓ **Partitioning**

$$38 \times 5 = (30 \times 5) + (8 \times 5)$$

$$= 150 + 40$$

$$= 190$$

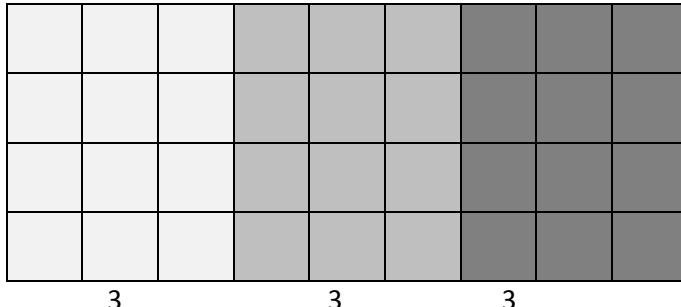
Encourage the children to be flexible about how they partition larger numbers into more useful arrays e.g.

9

4



Could be seen as



4

$$9 \times 4 = (3 \times 4) + (3 \times 4) + (3 \times 4)$$

$$= 12 + 12 + 12$$

To be successful in calculation, learners must have plenty of experiences of being flexible with portioning as this is the basis of the distributive and associative law.

Associative Law

e.g. $3 \times (3 \times 4) = 36$ The principle that if there are 3 numbers to multiply these can be multiplied in any order.

Distributive Law

e.g. $6 \times 14 = (2 \times 10) + (4 \times 10) + (4 \times 6) = 20 + 40 + 24 = 84$

This law allows you to distribute a multiplication across an addition or subtraction.

✓ Scaling

e.g. Find a ribbon that is 4 times as long as the blue ribbon



✓ Inverse Operations

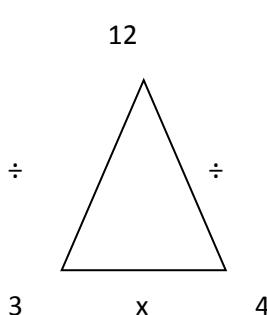
Trios can be used to model the 4 repeated multiplication and division facts. Children learn to state the related facts

$$3 \times 4 = 12$$

$$4 \times 3 = 12$$

$$12 \div 4 = 3$$

$$12 \div 3 = 4$$



Solving calculations with missing numbers can be supported by inverse operations or arrays

$$\square \times 5 = 20$$

$$3 \times \triangle = 18$$

$$\square \times \textcircled{O} = 32$$

2

$$2 \times ? = 12$$

Y4

Children will continue to use arrays and partitioning where appropriate to prepare them for the grid method of multiplication. Arrays can be represented as grids in a shorthand version. Place value counters can be used to show multiples of ten, hundred etc.

TO x O (Short multiplication – multiplication by a single digit)

23×8 Children will approximate first 23×8 is approximately $25 \times 8 = 200$

$$\begin{array}{r} \times \quad 20 \quad 3 \\ 8 \quad \boxed{160} \quad 24 \\ \hline \end{array} \quad \begin{array}{r} 160 \\ + \quad 24 \\ \hline 184 \end{array}$$

HTO x O (Short multiplication – multiplication by a single digit)

346×9 Children will approximate first 346×9 is approximately $350 \times 10 = 3500$

$$\begin{array}{r} \times \quad 300 \quad 40 \quad 6 \\ 9 \quad \boxed{2700} \quad 360 \quad 54 \\ \hline \end{array} \quad \begin{array}{r} 2700 \\ + \quad 360 \\ + \quad 54 \\ \hline 3114 \\ \quad \quad \quad 1 \ 1 \end{array}$$

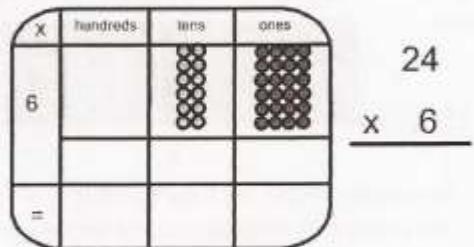
Formal Written Method

Short multiplication—multiplying by a single digit

The array using place value counters becomes the basis for understanding short multiplication first without exchange before moving onto exchanging

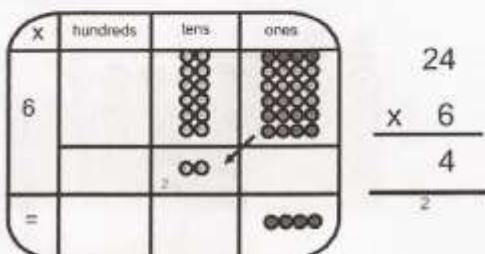
$$24 \times 6$$

1.



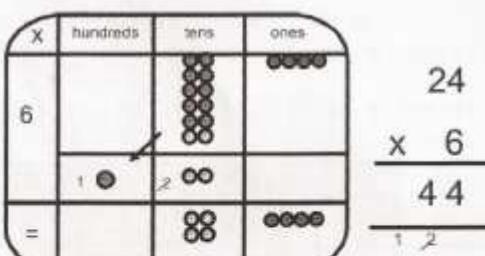
$$\begin{array}{r} 24 \\ \times 6 \\ \hline \end{array}$$

2.



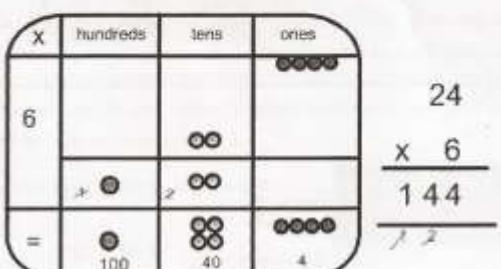
$$\begin{array}{r} 24 \\ \times 6 \\ \hline 4 \\ \hline 2 \end{array}$$

3.



$$\begin{array}{r} 24 \\ \times 6 \\ \hline 44 \\ \hline 12 \end{array}$$

4.



$$\begin{array}{r} 24 \\ \times 6 \\ \hline 144 \\ \hline 12 \end{array}$$

Y5

Grid method

(TO x TO) (Long multiplication – multiplication by more than a single digit)

72×38 Children will approximate first 72×38 is approximately $70 \times 40 = 2800$

x	70	2	
30	2100	60	2100
8	560	16	+ 560
			+ 60
			+ <u>16</u>
			<u>2736</u>
			1

Formal Written Method

Children should refer back to the grid method and compare before being required to record.

24	12
X 16	<u>124</u>
240 (24 x 10)	<u>X26</u>
<u>144</u> (24 x 6)	2480
384	<u>744</u>
2	<u>3224</u>
	11

Using similar methods, they will be able to multiply numbers up to 4 digits by a one or two digit number.

Y6

Using similar methods, they will be able to multiply decimals with up to two decimal places by a single digit number and then two digit numbers, approximating first. They should know that the decimal points line up under each other.

For example:

4.92×3 Children will approximate first 4.92×3 is approximately $5 \times 3 = 15$

x	4	0.9	0.02	
3	12	2.7	0.06	12
				+ 0.7
				+ 0.06
				<u>12.76</u>

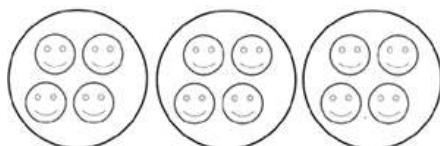
+ - + - + - + - + - +

By the end of year 6, children will have a range of calculation methods, mental and written. Selection will depend upon the numbers involved.

Chapter 5: PROGRESSION THROUGH CALCULATIONS FOR DIVISION

YR

Children are encouraged to develop a mental picture of the number system in their heads to use for calculation. They will experience equal groups and sharing items using a wide variety of equipment, (e.g. small world play, role play, counters, cubes etc.) in both play and problem solving contexts. They will count in 2's 10's and 5's. They will develop ways of recording calculations using pictures, etc.



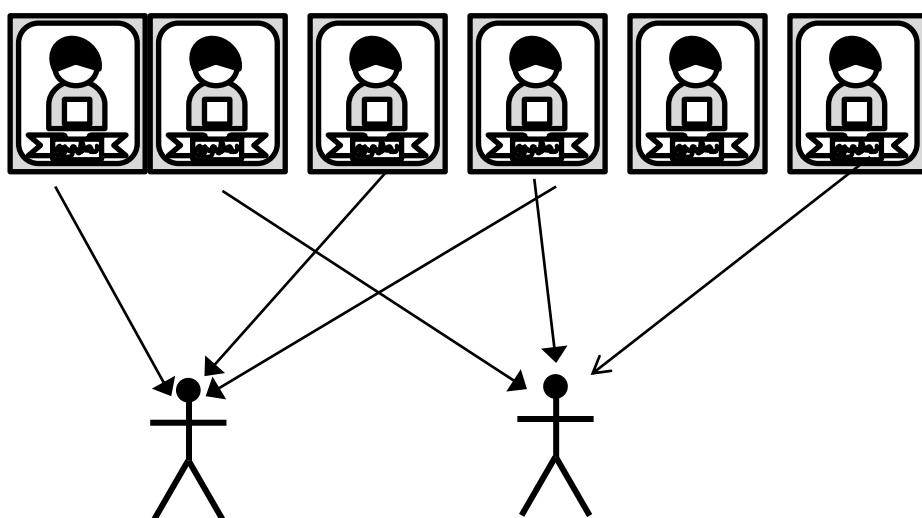
Y1

Sharing

In problem solving contexts, children will use practical equipment to share out objects equally and to group objects to represent division.

a) 6 football stickers are shared between 2 people, how many do they each get?

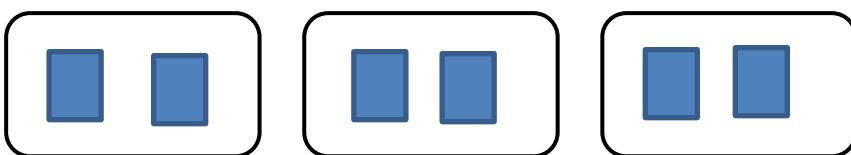
Children may solve this by using a 'one for you, one for me' strategy until all of the cards have been given out.



Children find the answer by counting how many cards **1 person** has got.

Grouping or Repeated Subtraction

There are 6 football stickers, how many people can have 2 stickers each?



Children find the answer by counting how many **groups of 2** there are.

The teacher can model the link between sharing and grouping in the following way by relating back to the first football sticker question:

Placing the football stickers in a bag or box, the teacher can ask the children how many stickers would need to be taken out of the box to give each person one sticker each (i.e. 2) and exemplify this by putting the cards in groups of 2 until all cards have been removed from the bag.

Y2

Children will utilise practical equipment to represent division calculations as grouping (repeated subtraction) and use jottings to support their calculation, e.g.

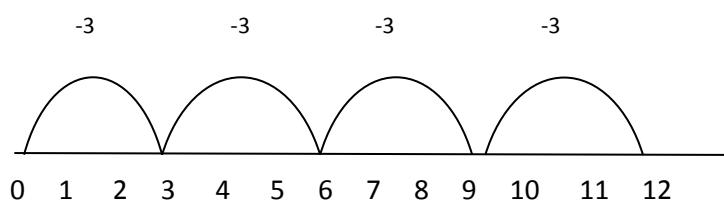
Bead string

$$12 \div 3 =$$



Children need to understand that this calculation reads as 'How many groups of 3 are there in 12?'

Numberline



The number line can also be used to model division through repeated subtraction.

Numicon and Cuisenaire



Children also move onto calculations involving remainders.

$$13 \div 4 =$$



$$13 \div 4 = 3 \text{ remainder } 1$$

Arrays

Children learn to model a division calculation using an array. This model supports their understanding of the development of partitioning and the ‘bus stop’ method in a written method. This method also connects division to finding fractions of discrete quantities

$$\begin{array}{ccccc} \circ & \circ & \circ & \circ & \circ \\ \circ & \circ & \circ & \circ & \circ \\ \circ & \circ & \circ & \circ & \circ \end{array} \quad 15 \div 3 = 5$$
$$15 \div 5 = 3$$

Recording

Children will record their calculations in the form $28 \div 7 = 4$ or $28 \div 6 = 4 \text{ r } 4$

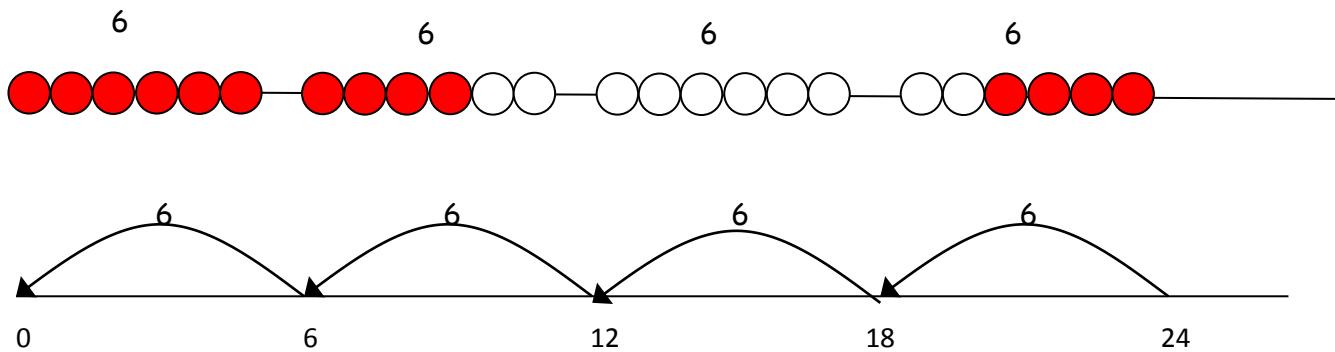
Y3

Children will continue to use:

✓ **Grouping or Repeated Subtraction**

$24 \div 6$ (24 marbles, how many people can have 6 marbles each)

This should be linked to the number line to reinforce that grouping is repeated subtraction.



✓ **Inverse Operations**

Trios can be used to model the 4 repeated multiplication and division facts. Children learn to state the related facts (see multiplication chapter)

Children can use arrays or inverse operations to complete equations where symbols stand for unknown numbers.

$$24 \div 2 = \quad 15 \div \square = 3 \quad \triangle \div 10 = 8$$

Children use knowledge of inverse operations to complete missing number problems

This can also be supported with arrays

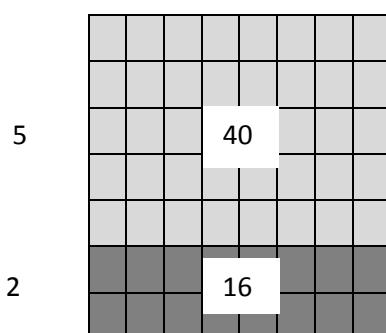
$$\begin{array}{|c|c|c|c|} \hline 12 & & & \\ \hline \end{array} \quad 12 \div 2 = ?$$

2

✓ **Arrays and Partitioning**

The array is a flexible model for division of larger numbers

$$56 \div 8 = 7 \quad \begin{array}{|c|c|c|c|c|c|c|c|} \hline 8 & & & & & & & \\ \hline & & & & & & & \\ \hline & & & & & & & \\ \hline & & & & & & & \\ \hline & & & & & & & \\ \hline & & & & & & & \\ \hline & & & & & & & \\ \hline & & & & & & & \\ \hline \end{array}$$



Children could break this down into more manageable arrays as well as using their understanding of the inverse relationship between division and multiplication.

$$56 \div 8 = (40 \div 8) + (16 \div 8) = 5 + 2 = 7 \quad (\text{Distributive Law – a division can be distributed across an addition or subtraction})$$

Remainders

Children need to be able to decide what to do with remainders after division and round up or down accordingly. They make sensible decisions about rounding up or down after division. For example $62 \div 8$ is 7 remainder 6, but whether the answer be rounded up to 8 or rounded down to 7 depends on the context.

e.g. I have 62p. Sweets are 8p each. How many can I buy?

Answer: 7 (the remaining 6p is not enough to buy another sweet)

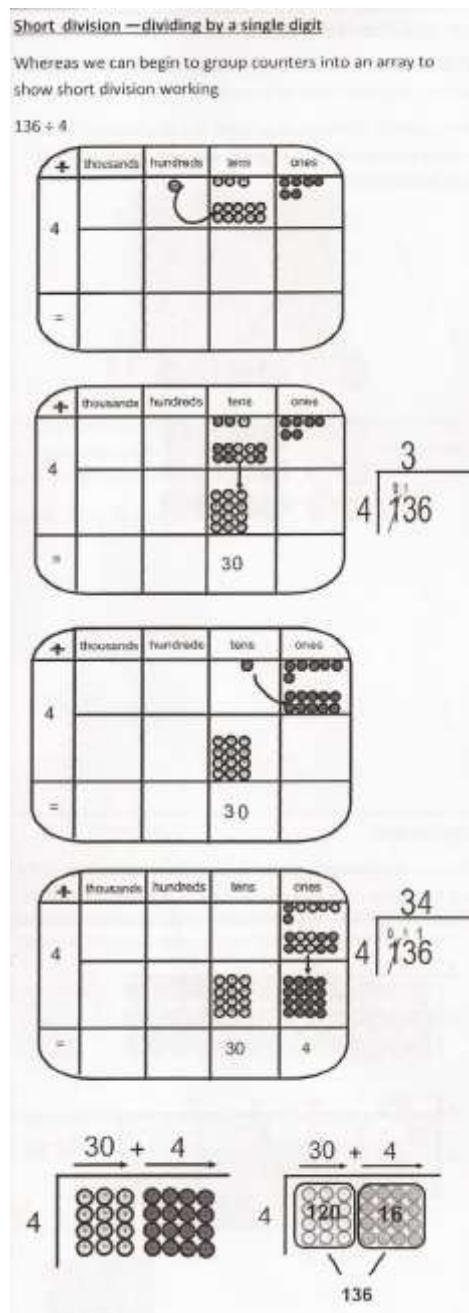
Apples are packed into boxes of 8. There are 62 apples. How many boxes are needed?

Answer: 8 (the remaining 6 apples still need to be placed into a box)

Y4

Children will develop their use of grouping (repeated subtraction) to be able to subtract multiples of the divisor, developing the use of the 'chunking' method.

Short division (TO ÷ O)



$$\begin{array}{r}
 & 3 \ 4 \ r \ 1 \\
 4) & 1 \ 3 \ 7 \\
 - & 1 \ 2 \\
 \hline
 & 1 \ 7 \\
 - & 1 \ 6 \\
 \hline
 & 1
 \end{array}
 \quad
 \begin{array}{r}
 4 \times 3 \\
 \hline
 4 \times 4
 \end{array}$$

Ans 34 r 1

Key facts
 $x2 \ 8$
 $x5 \ 20$
 $x10 \ 40$

Children can write key facts in a menu box. This will help them in identifying the largest group they can subtract in one chunk

Children write their answer above the calculation to make it easy for them and the teacher to distinguish.

Any remainders should be shown as integers, i.e. 14 remainder 2 or 14 r 2.

Y5

Children can start to subtract larger multiples of the divisor (e.g. 20x).

Short division (HTO ÷ O, ThHTO)

$$136 \div 4$$

$$\begin{array}{r} 34 \\ 4 \overline{)136} \\ -120 \\ \hline 16 \\ -16 \\ \hline \end{array}$$

30x4
4 x 4

Answer : 34

Any remainders should be shown as integers, i.e. 14 r 2.

Children need to be able to decide what to do after division and round up or down accordingly. They make sensible decisions about rounding up or down after division. For example $240 \div 52$ is 4 remainder 32, but whether the answer should be rounded up to 5 or rounded down to 4 depends on the context.

Y6

Long division (HTO ÷ TO)

$$432 \div 15$$

$$\begin{array}{r} 28 \\ 15) \overline{432} \\ -300 \quad 20 \times 15 \\ \hline 132 \\ -120 \quad 8 \times 15 \\ \hline 12 \\ \downarrow \\ \text{Answer } 28 \text{ r } 12 \end{array}$$

Any remainders should be shown as fractions i.e. $432 \div 15 = 28 \text{ r } 12$ should be shown as $28 \frac{12}{15}$ which is written $28 \frac{4}{5}$ in its lowest terms.

Or extend the method for decimals up to two decimal places. Children know that decimal points line up under each other.

$$\begin{array}{r} 2.8.8 \\ 15) \overline{432.0} \\ -30 \downarrow \\ 132 \\ -120 \downarrow \\ 120 \\ -120 \\ \hline 0 \end{array}$$

Answer 28.8

+ - + - + - + - + - +

By the end of year 6, children will have a range of calculation methods, mental and written. Selection will depend upon the numbers involved.